Pergamon

PII: S0191–8141(98)00027–3

Journal of Structural Geology, Vol. 20, No. 7, pp. 975 to 977, 1998

0191-8141/98 $19.00 + 0.00

BREVIA

Forward modeling of non-steady-state deformations and the 'minimum strain path': Discussion

DAZHI JIANG

Department of Geology, University of New Brunswick, Fredericton, NB, Canada E3B 5A3

(*Received* 5 *November* 1997; *accepted in revised form* 1 *December* 1997)

INTRODUCTION

It is well known that there is an infinite number of possible deformation paths that can bring a volume of rock from one configuration to another. If the deformation is homogeneous, one of these paths is steady state (Passchier, 1990). All of these paths are, by definition, 'equal-strain paths' because they all start with the same initial configuration and end up with the same final configuration. Different paths, however, differ in their efficiency of accumulating finite strain (McKenzie, 1979; Means *et al.*, 1980; Pfiffner and Ramsay, 1982). Although a deformation path can be described kinematically, what actually govern a volume of rock to follow certain path(s) are dynamic laws (see Jiang and White, 1995, for some discussion).

By defining a very unusual 'offset' and setting the deformation to achieve it, Fossen and Tikoff (1997) argued for the 'minimum strain path', and suggested that certain natural deformations might follow such paths. This discussion attempts to demonstrate that the minimum strain path arises solely from mathematical manipulation. Depending on the kinematic quantity one wishes to prescribe, there are an infinite number of possible 'minimum strain paths'. None of them correspond to the real physical process of natural deformation.

Because Fossen and Tikoff's (1997) argument is based on their use of a very unusual 'offset', I first discuss different measures of shear zone displacement.

MEASURE OF DISPLACEMENT ACROSS A SHEAR ZONE

The 'offset' defined by Fossen and Tikoff (1997, p. 989 and their fig. 2) is very unusual. It is the shear-zone-parallel component of the displacement of the material particle initially at the 'upper, right-hand corner'. Figure 1 compares their 'offset' (U) with the commonly used measure of a shear zone displacement (δ) as indicated by the displaced 'dykes' initially perpendicular to the shear zone. U differs from δ unless the deformation path is a simple shear (Fig. 1). Although U = 2 for all situations in fig. 2 of Fossen and Tikoff (1997), the δ-values are different: $\delta = 2$ for the simple shear case (fig. 2a), $\delta = 0$ for the pure shear case (fig. 2b) and $\delta \approx 1.4$ for the sub-simple shear case ($W_k = 0.82$, fig. 2c).

U and δ can be easily related by considering the displacement field of the deformation.

An arbitrary material particle initially at position $P(X, Y)$ is transformed to a new position $p(x, y)$ by deformation (Fig. 1). The displacement is:

$$u = x - X, \quad v = y - Y, \tag{1}$$

where u and v are the displacement components along the horizontal and vertical axes, respectively (Fig. 1). For the deformation relevant here, (x, y) and (X, Y) are related by a position gradient tensor of the form:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & \delta \\ 0 & a^{-1} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}, \tag{2}$$

where a is the stretch parallel to the shear zone.

Combining (1) and (2) gives:

$$u = (a - 1)X + \delta Y, \quad v = (a^{-1} - 1)Y. \tag{3}$$

Considering a shear zone of unit thickness in the undeformed state (this means U and δ are quantities normalized against the initial thickness of the shear zone) without losing any generality, the particle initially at 'the upper, right-hand corner' has an initial position: $(X, Y) = (1, 1)$ (Fig. 1). Using (3), the relationship between U and δ is:

$$U = (a - 1) + \delta. \tag{4}$$

Although very unusual, U is a valid measure of shear zone displacement. In addition to δ and U, a third possible measure of the shear zone displacement is the shear

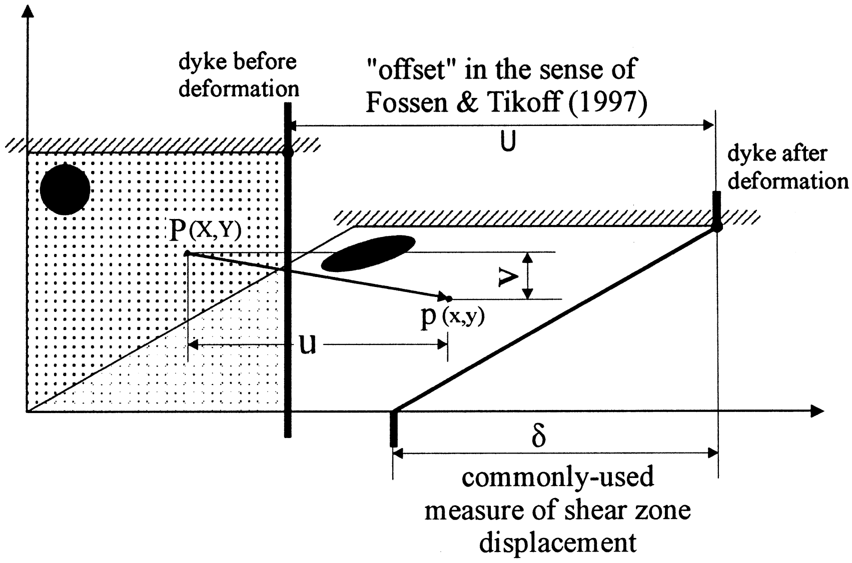

Fig. 1. The difference between the 'offset' (U) used by Fossen and Tikoff (1997) and the commonly used measure of shear zone displacement (δ). The shear zone deformation transforms an arbitrary material particle from $P(X, Y)$ to $p(x, y)$. The displacement P–p has a horizontal component U and a vertical component V. See text for details.

strain on the shear zone boundary γ ($= a\delta$), which is the actual shear zone displacement normalized against the shear zone thickness in the deformed state. In fact, so long as it is a mathematically positive and monotonically-increasing function of δ, or both δ and a, any quantity can be regarded as some measure of the shear zone displacement. But it is important to realize that the displacement measures (δ, γ, U or others) are not only functions of time but also functions of the path.

AN INFINITE NUMBER OF 'MINIMUM STRAIN PATHS'

Figure 2 plots the finite strain state ($R_f = \lambda_1^{1/2}/\lambda_2^{1/2}$) for different paths against different displacements [Fig. 2a: $R_f = R_f(W_k, \delta)$, Fig. 2b: $R_f = R_f(W_k, \gamma)$ and Fig. 2c: $R_f = R_f(W_k, U)$]. From each R_f–displacement relationship, one can calculate the 'minimum strain path'. The steady-state minimum strain paths* for δ, γ and U are plotted in Fig. 3. For prescribed δ of any value, the minimum strain path is a simple shear. For prescribed shear strain γ of values: 0, 1, 2, 3, 4, 5 and 8, the minimum strain paths are, respectively, $W_k = 1$, 0.94, 0.85, 0.81, 0.79, 0.77 and 0.75. For prescribed U of values of 0, 2, 5 and 8, the minimum strain paths are, respectively, $W_k = 0.89$, 0.82, 0.74 and 0.72 (Fossen and Tikoff, 1997, p. 990).

One can potentially set any other kinematic quantities as prescribed, and solve for the minimum strain paths to achieve them. The paths are different depending on the quantity prescribed as shown in Fig. 3. Therefore there exists an infinite number of different 'minimum strain paths'; it would make no sense to speak of the minimum strain path unless the prescribed kinematic quantity is specified.

Another point to make here is that one must be careful when reading plots like Fig. 2. In Fig. 2, R_f is plotted for different paths (different curves) against displacements which are themselves path dependent, as pointed out earlier. This drawback obscures the physical significance of the curves, and could make the plot misleading. For example, the fact that the curves for sub-simple shear paths are *below* the curves for the simple shear path in Fig. 2(c) (and fig. 3 of Fossen and Tikoff, 1997) could easily mislead one to think that sub-simple shear will accumulate finite strain more *slowly* than does simple shear. 'Slowly' refers to time-rate only, but the horizontal axes of the curves are 'displacements' which are dependent on path also.

CONCLUSIONS

A minimum strain path arises only when a kinematic quantity is prescribed and the deformation is set to achieve it. There are an infinite number of possible minimum strain paths, depending on the quantity prescribed. A volume of rock certainly does not know which one to follow. Being an energetic process, natural deformation of rocks is expected to follow paths that are both feasible kinetically and most 'economic' energetically (the principle of least action) (cf. Goods and Brown, 1979). The comparison between the minimum work path and the 'minimum strain path' (Fossen and Tikoff, 1997) is inadequate, because the former is concerned with the dynamics of a defor-

*By making the shear strain increment infinitesimal, the non-steady-state minimum strain path can be constructed. This is not done here, because it is already sufficient to make the point with the steady-state minimum strain path.

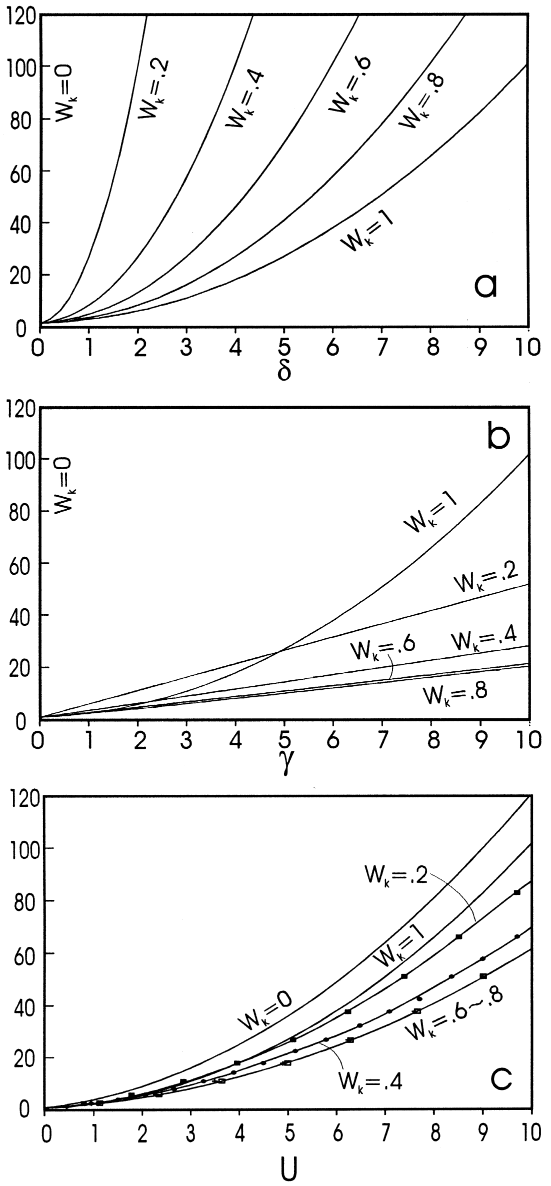

Fig. 2. Plot of the finite strain ratio ($R_f=\lambda_1^{1/2}/\lambda_2^{1/2}$) for different deformation paths against different displacements. (a) R_f–δ plot, (b) R_f–γ plot, and (c) R_f–U plot. See text for details.

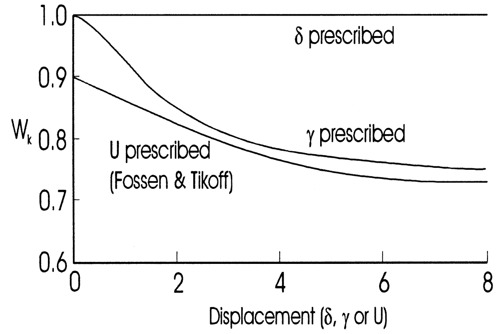

Fig. 3. Different steady-state minimum strain paths arise when different kinematic quantities are prescribed. See text for details.

mation whereas the latter arises from a purely geometric curiosity. It is impossible for an energetic process to follow a path that is determined purely kinematically.

Acknowledgements—Advice and comments from Sue Treagus and Cees Passchier helped me to present this discussion more clearly. I also thank Shoufa Lin, Ron Pickerill, Peter Stringer and Paul Williams for discussion and suggestions.

REFERENCES

Fossen, H. and Tikoff, B. (1997) Forward modeling of non-steady-state deformations and the 'minimum strain path'. *Journal of Structural Geology* **19**, 987–996.

Goods, S. H. and Brown, L. M. (1979) The nucleation of cavities by plastic deformation. *Acta Materiala* **27**, 1–15.

Jiang, D. and White, J. C. (1995) Kinematics of rock flow and the interpretation of geological structures, with particular reference to shear zones. *Journal of Structural Geology* **17**, 1249–1265.

McKenzie, D. (1979) Finite deformation during fluid flow. *Geophysical Journal of the Royal Astronomical Society* **58**, 689–715.

Means, W. D., Hobbs, B. E., Lister, G. S. and Williams, P. F. (1980) Vorticity and non-coaxiality in progressive deformations. *Journal of Structural Geology* **2**, 371–378.

Passchier, C. W. (1990) A Mohr circle construction to plot the stretch history of material lines. *Journal of Structural Geology* **12**, 513–515.

Pfiffner, O. A. and Ramsay, J. G. (1982) Constraints on geological strain rates: arguments from finite strain states of naturally deformed rocks. *Journal of Geophysical Research* **87**, 311–321.